

Level Structure of Bi^{208} as Observed at High Resolution with the $\text{Bi}^{209}(d,t)\text{Bi}^{208}$ Reaction*

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The $\text{Bi}^{209}(d,t)\text{Bi}^{208}$ reaction has been studied at a bombarding energy of 12 MeV with a high-resolution magnetic spectrograph. A ground-state Q value of -1.201 ± 0.005 MeV was measured. Excited states were observed at 0.063, 0.513, 0.605, 0.634, 0.651, 0.890, 0.929, 0.963, 1.074, 1.098, 2.349, 2.394, and 2.417 MeV. The angular-distribution data taken separate the $h_{9/2}p_{3/2}^{-1}$ states from the $h_{9/2}f_{5/2}^{-1}$ states. Spin assignments based on the observed cross sections of the levels are given. In particular, the ground state and first excited state are assigned $J=5$ and $J=4$, respectively. The resulting interpretation is in good agreement with the Bi^{208} calculations by Kim and Rasmussen.

INTRODUCTION

THE energy levels in the odd-odd nucleus Bi^{210} have recently been studied¹ with the $\text{Bi}^{209}(d,p)\text{Bi}^{210}$ reaction at high resolution. Kim and Rasmussen² were able to obtain a good fit to the low-lying excited states in Bi^{210} by use of a jj -coupled odd-group model, with configuration mixing and a residual nucleon-nucleon interaction that included tensor forces. The interaction that they chose was a slightly modified form of the free-nucleon interaction. Mello and Flores³ have also obtained a good fit to the low-lying excited states in Bi^{210} . This success for Bi^{210} encourages the hope for equally successful interpretations for the other odd-odd nuclei adjacent to the doubly-magic nucleus Pb^{208} . Hopefully, the residual nucleon-nucleon interaction would stay the same for these other nuclei.

Kim and Rasmussen⁴ have recently calculated the energy levels in Bi^{208} by use of the same residual interaction that was used in their calculations of Bi^{210} . The excitation energies and wavefunctions which they obtained are given in Tables I and II. Previously, Wahlborn⁵ had calculated the positions of low-lying levels in Bi^{208} . His calculations predicted that the ground state has $J^\pi = 4^+$. These spin assignments are the reverse of the results of Kim and Rasmussen.

Experimental information about the energy levels of Bi^{208} has been obtained by radioactive decay methods and by studies with charged-particle reactions. The decay experiments⁶ have produced evidence for five energy levels. The first to study Bi^{208} by a charged-particle reaction was Harvey,⁷ who used the $\text{Bi}^{209}(d,t)\text{Bi}^{208}$ re-

action. He measured a ground-state Q value of -1.17 MeV for this reaction and found groups of excited states at 0.59- and 1.01-MeV excitation. More recently the $\text{Bi}^{209}(d,t)\text{Bi}^{208}$ reaction was studied first by Cohen *et al.*,⁸ who used about a 100-keV resolution width, and then by Mukherjee and Cohen⁹ with a 45-keV resolution width. The energy levels of Bi^{208} given in the latter two reports do not agree with each other even when an allowance is made for the differences in resolution. Furthermore, the level structure reported in the higher resolution study is in serious disagreement with the calculations of Kim and Rasmussen. The present work was undertaken to provide better information concerning the energy levels of Bi^{208} .

EXPERIMENTAL PROCEDURE

A broad-range magnetic spectrograph was used to record the reaction data. This instrument is similar to the original magnetic spectrograph at MIT,¹⁰ except that the radius of curvature is 1.52 times as large. This larger size permits the study of 45-MeV protons or of other particles with an equivalent magnetic rigidity. The ion optics are the same as in the MIT spectrograph. Nuclear track plates are used to record the analyzed particles. Both the number of exposures which can be taken in one loading and the maximum scattering angle available have been increased. The target chamber has a sliding seal¹¹ which permits the scattering angle to be changed while the chamber is under a vacuum. Also, a quadrupole lens improves the solid angle of the instrument when it is operated as a spectrometer.¹² The spectrograph was calibrated with alpha particles from a Po^{210} source prepared in the manner described by Browne *et al.*¹³ The alpha-particle energy was taken to be 5.3045 MeV. The resolution width of the

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¹ J. R. Erskine, W. W. Buechner, and H. A. Enge, *Phys. Rev.* **128**, 720 (1962).

² Y. E. Kim and J. O. Rasmussen, *Nucl. Phys.* **47**, 184 (1963).

³ P. A. Mello and J. Flores, *Nucl. Phys.* **47**, 177 (1963).

⁴ Y. E. Kim and J. O. Rasmussen, *Phys. Rev.* **135**, B44 (1964).

⁵ S. Wahlborn, *Nucl. Phys.* **3**, 644 (1957).

⁶ I. Perlman, F. Asaro, F. S. Stephens, J. P. Hummel, and R. C. Pilger, University of California Radiation Laboratory Report UCRL-2932, 1955, p. 59 (unpublished); R. B. Duffield and S. H. Vegors, Jr., *Phys. Rev.* **112**, 1958 (1958); V. L. Glagolev, P. A. Yampolskii, *Zh. Eksperim. i. Teor. Fiz.* **40**, 743 (1961) [English transl.: *Soviet Phys.—JETP* **13**, 520 (1961)]; W. B. Jones, *Phys. Rev.* **130**, 2042 (1963).

⁷ J. A. Harvey, *Can. J. Phys.* **31**, 278 (1953).

⁸ B. L. Cohen, S. Mayo, and R. E. Price, *Nucl. Phys.* **20**, 360 (1960).

⁹ P. Mukherjee and B. L. Cohen, *Phys. Rev.* **127**, 1284 (1962).

¹⁰ C. P. Browne and W. W. Buechner, *Rev. Sci. Instr.* **27**, 899 (1956).

¹¹ J. R. Erskine and C. P. Browne, *Nucl. Instr. Methods* **13**, 359 (1961).

¹² H. A. Enge, *Rev. Sci. Instr.* **29**, 885 (1958).

¹³ C. P. Browne, J. A. Galey, J. R. Erskine, and K. L. Warsh, *Phys. Rev.* **120**, 905 (1960).

TABLE I. Excitation energies of the levels in Bi²⁰⁸ as calculated by Kim and Rasmussen.

Configuration ^a	J ^π	Excitation energy (MeV)
<i>h</i> _{9/2} <i>p</i> _{1/2} ⁻¹	4 ⁺	0.081
	5 ⁺	0.0
	2 ⁺	0.920
<i>h</i> _{9/2} <i>f</i> _{5/2} ⁻¹	3 ⁺	0.630
	4 ⁺	0.596
	5 ⁺	0.622
	6 ⁺	0.529
	7 ⁺	0.664
	3 ⁺	1.046
<i>h</i> _{9/2} <i>p</i> _{3/2} ⁻¹	4 ⁺	0.981
	5 ⁺	0.916
	6 ⁺	1.079
	3 ⁺	0.988
<i>f</i> _{7/2} <i>p</i> _{1/2} ⁻¹	4 ⁺	1.060
	1 ⁺	2.124
<i>f</i> _{7/2} <i>f</i> _{5/2} ⁻¹	2 ⁺	1.701
	3 ⁺	1.669
	4 ⁺	1.705
	5 ⁺	1.555
	6 ⁺	2.019
	2 ⁺	2.183
<i>f</i> _{7/2} <i>p</i> _{3/2} ⁻¹	3 ⁺	1.960
	4 ⁺	1.889
	5 ⁺	1.975
	1 ⁺	2.850
	2 ⁺	2.531
<i>h</i> _{9/2} <i>f</i> _{7/2} ⁻¹	3 ⁺	2.482
	4 ⁺	2.469
	5 ⁺	2.417
	6 ⁺	2.484
	7 ⁺	2.373
	8 ⁺	2.637
	0 ⁺	4.289
	1 ⁺	4.286
	2 ⁺	3.566
	3 ⁺	3.536
<i>f</i> _{7/2} <i>f</i> _{7/2} ⁻¹	4 ⁺	3.357
	5 ⁺	3.430
	6 ⁺	3.260
	7 ⁺	3.518
	2 ⁻	2.755
	3 ⁻	1.803
	4 ⁻	1.990
	5 ⁻	1.822
	6 ⁻	1.847
	7 ⁻	1.841
<i>h</i> _{9/2} <i>i</i> _{13/2} ⁻¹	8 ⁻	1.763
	9 ⁻	1.907
	10 ⁻	1.687
	11 ⁻	2.239
	3 ⁻	3.055
	4 ⁻	2.648
	5 ⁻	2.627
	6 ⁻	2.595
	7 ⁻	2.545
	8 ⁻	2.593
<i>f</i> _{7/2} <i>i</i> _{13/2} ⁻¹	9 ⁻	2.505
	10 ⁻	2.679

^a In this notation, the proton state is written on the left and the neutron-hole state is on the right.

spectrograph is smaller than 0.1% in energy at a solid angle of 3 × 10⁻⁴ sr. The energy calibration is reproducible to better than 0.1%.

The deuteron beam was provided by a High Voltage Engineering Corporation Model EN tandem accelerator. At an energy of 12 MeV, beam currents of 1 μA are available with the beam analyzer slits set at a geometrical resolution width of 0.1% in energy.

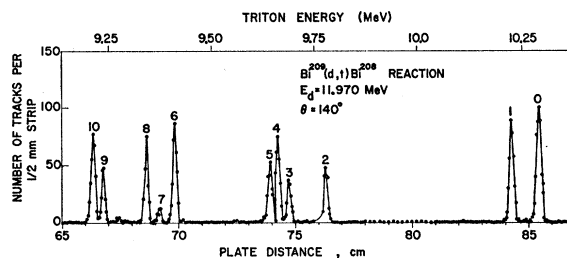


FIG. 1. Spectrum of tritons observed at 140° from a bismuth target bombarded with 12.0-MeV deuterons.

The targets were made by evaporating natural bismuth metal on self-supporting carbon backings. The backings were purchased from Oak Ridge National Laboratory, Stable Isotopes Division.

Three exposures of about 3000 μC each were made to observe the Bi²⁰⁹(d,t)Bi²⁰⁸ reaction at scattering angles of 60°, 100°, and 140°. Shorter exposures were made to record the elastically scattered deuterons for the measurement of bombarding energy, target composition, and target thickness. A bombarding energy of about 12 MeV was used for all the exposures. A solid-state detector was used as a monitor counter during the exposures to eliminate problems in integrating the beam current and to compensate for variations in target thickness. A single-channel pulse-height analyzer was set on the elastic-scattering peak of bismuth.

The absolute differential cross sections for the Bi²⁰⁹(d,t)Bi²⁰⁸ reaction were obtained by comparing the yield of the (d,t) reaction with the yield of deuterons that had been elastically scattered off bismuth nuclei at a 30° scattering angle. An optical-model calculation had shown that for 12-MeV deuterons elastically scattered from bismuth, the ratio of actual scattering cross section to Rutherford scattering cross section stays within a few percent of unity out to a scattering angle of about 40°. Consequently, the assumption was made that at a scattering angle of 30° the elastic-scattering cross section was pure Rutherford scattering.

EXPERIMENTAL RESULTS

The triton spectrum recorded at a scattering angle of 140° is shown in Fig. 1. Ten groups which correspond to levels in Bi²⁰⁸ are present. The background is practically zero. The three very weak groups which corresponded to excited states near 2.5 MeV are not shown. The ground-state Q value of the Bi²⁰⁹(d,t)Bi²⁰⁸ reaction was measured to be -1.201 ± 0.005 MeV. The excitation energies and Q values calculated from the data as well as suggested spin and configuration assignments are given in Table III. The absolute differential cross section measured for the various groups at scattering angles of 60°, 100°, and 140° are listed in Table IV. The absolute values of these differential cross-section measurements are accurate to about 10% at one observation angle. The accuracy of the cross sections relative to

TABLE II. Wave functions of the Bi²⁰⁸ energy levels as calculated by Kim and Rasmussen.

J	Excitation energies (MeV)	Wave functions								
		$h_{9/2}p_{1/2}$	$h_{9/2}f_{5/2}$	$h_{9/2}p_{3/2}$	$f_{7/2}p_{1/2}$	$f_{7/2}f_{5/2}$	$f_{7/2}p_{3/2}$	$h_{9/2}f_{7/2}$	$f_{7/2}f_{7/2}$	
2	0.920		0.9872				-0.0215	-0.0156	-0.1568	-0.0122
3	0.630		0.9368	0.3471	0.0396	0.0001	0.0137	0.0135	0.0038	
	0.988		-0.0251	-0.0372	0.9760	-0.1213	-0.1706	0.0195	-0.0350	
	1.046		-0.3438	0.9303	0.0281	0.0373	-0.0287	-0.1148	-0.0133	
4	0.081	0.9763	-0.1806	-0.1128	-0.0025	-0.0034	-0.0019	-0.0374	-0.0059	
	0.596	0.1487	0.9594	-0.2339	0.0220	-0.0002	-0.0109	-0.0462	-0.0046	
	0.981	0.1454	0.2080	0.9603	-0.0582	-0.0108	-0.0152	-0.0977	-0.0102	
	1.060	0.0074	-0.0087	0.0581	0.9729	-0.1766	0.1358	-0.0193	0.0166	
5	0.0	0.9806	-0.1801	0.0739		-0.0071	0.0011	0.0203	0.0027	
	0.622	0.1532	0.9496	0.2713		-0.0088	0.0128	0.0320	0.0055	
	0.916	-0.1175	-0.2530	0.9587		0.0237	-0.0007	-0.0506	-0.0021	
6	0.529		0.9920	-0.1235		0.0047		-0.0250	-0.0016	
	1.079		0.1200	0.9861		-0.0056		-0.1149	-0.0087	
7	0.664		0.9992					0.0404	0.0018	

each other is greater since it is determined primarily by the statistical fluctuations in the number of tracks in each group. Table V lists the number of tracks observed in each group. From this table it can be seen, for example, that the uncertainty in the relative cross section of the ground-state doublet observed at a scattering angle of 100° is about 5%.

DISCUSSION

The levels in Bi²⁰⁸ observed in the present work and/or by earlier workers, are shown in Fig. 2. Only the ground-state doublet observed in the present work agrees with the level scheme of Mukherjee and Cohen. An examination of the level schemes shows that the agreement is improved considerably if the higher ex-

cited states reported by Mukherjee and Cohen are shifted upward by about 0.25 MeV. This might be a coincidence, but it suggests that their ground-state group was misidentified. The coarse resolution experiments of Cohen *et al.* showed unresolved groups at excitations of 0.49, 0.61, 0.87, 1.05, 1.65, 2.34, and 2.60 MeV, in reasonable agreement with the clusters of levels observed in the present experiment. Harvey's early (d,t) results are similar to those of Cohen *et al.* The five levels of Bi²⁰⁸ observed in the radioactive decay seem to correspond to some of the levels observed in the present work. In particular, the spin assignment and energy of the 510-keV level reported in the decay work are consistent with the present experiment. The level at 1.43-MeV excitation, not observed in the present (d,t) study, is possibly from an excited proton configuration which would be difficult to excite with this reaction.

The predicted spectrum of tritons from the Bi²⁰⁹(d,t)-Bi²⁰⁸ reaction was constructed from the excitation en-

TABLE III. Excitation energies, Q values, and suggested spins and configurations for the Bi²⁰⁸ levels formed through the Bi²⁰⁹(d,t)Bi²⁰⁸ reaction.

Level	E_x^a (MeV)	Q^b (MeV)	Suggested J	Suggested configuration ^c
0	0	-1.201	5	$h_{9/2}p_{1/2}^{-1}$
1	0.063	-1.264	4	
2	0.513	-1.714	6	$h_{9/2}f_{5/2}^{-1}$
3	0.605	-1.806	4	
4	0.634	-1.835	3 and 5	
5	0.651	-1.852	7	
6	0.890	-2.091	5	$h_{9/2}p_{3/2}^{-1}$
7	0.929	-2.130	2	$h_{9/2}f_{5/2}^{-5}$
8	0.963	-2.164	4	$h_{9/2}p_{3/2}^{-1}$
9	1.074	-2.275	3	
10	1.098	-2.299	6	
11	2.349	-3.550		$h_{9/2}f_{7/2}^{-1}$
12	2.394	-3.595		
13	2.417	-3.618		

^a The estimated uncertainty is 1 keV for level No. 1, 3 keV for levels Nos. 2-10, and 10 keV for levels Nos. 11-13.

^b The estimated uncertainty is 5 keV for levels Nos. 0-10 and 10 keV for levels Nos. 11-13.

^c In this notation, the proton state is on the left and the neutron-hole state is on the right.

TABLE IV. Differential cross sections for the various energy levels of Bi²⁰⁸ which were observed with the Bi²⁰⁹(d,t)Bi²⁰⁸ reaction.

Level	$d\sigma/d\Omega^a$		
	$\theta=60^\circ$ (mb/sr)	$\theta=100^\circ$ (mb/sr)	$\theta=140^\circ$ (mb/sr)
0	0.339	0.706	0.522
1	0.259	0.554	0.395
2	0.040	0.178	0.203
3	0.039	0.141	0.149
4	0.057	0.232	0.309
5	0.052	0.198	0.231
6	0.090	0.329	0.342
7	0.006	0.052	0.047
8	0.060	0.196	0.229
9	0.038	0.125	0.159
10	0.078	0.264	0.311
11			0.014
12			0.018
13			0.012

^a The estimated uncertainty in these quantities as absolute differential cross sections is about 10%. The uncertainty of these numbers relative to each other is somewhat less and depends primarily on the statistical fluctuations in the number of tracks in each group.

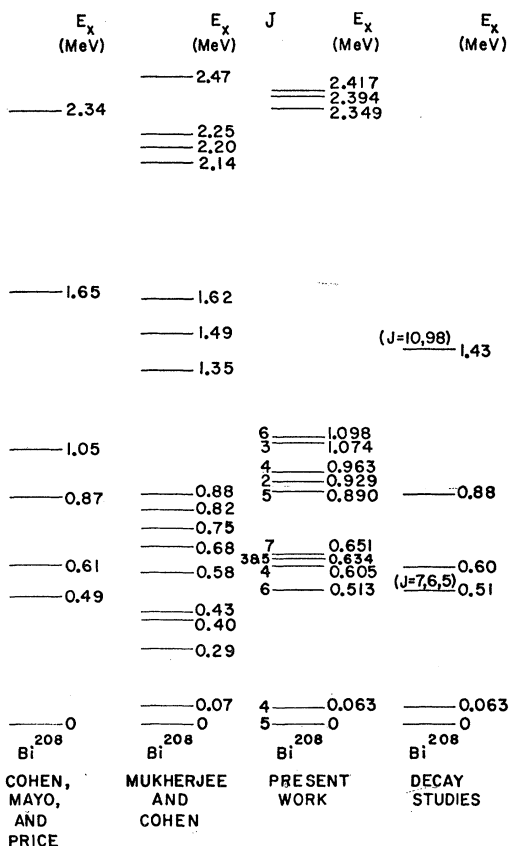


FIG. 2. Energy levels in Bi²⁰⁸ observed in the present work together with levels reported in earlier studies. The excitation energies and suggested spins are shown.

ergies (Table I) calculated by Kim and Rasmussen for Bi²⁰⁸ and the intrinsic single-particle cross sections which were calculated by the distorted-wave Born approximation (DWBA) code SALLY.¹⁴ The wave functions of

TABLE V. The number of tracks in each group recorded by the spectrograph at observation angles of 60°, 100°, and 140°.

Level	Number of tracks in each group		
	$\theta=60^\circ$	$\theta=100^\circ$	$\theta=140^\circ$
0	588	774	494
1	452	610	377
2	72	202	199
3	70	161	147
4	103	265	305
5	94	226	228
6	163	382	342
7	11	61	47
8	110	228	232
9	71	147	163
10	147	311	317
11			15
12			20
13			13

¹⁴ R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report ORNL-3240 (UC-34-Physics) (unpublished).

TABLE VI. Optical parameters used in the DWBA calculation.

Parameter	Deuteron	Triton
Potential option	7	7
V (MeV)	90.0	98.0
W (MeV)	0.0	11.0
$r_0(F)$	1.15	1.07
$r_c(F)$	1.15	1.40
$a(F)$	0.87	0.84
$r_0'(F)$	1.37	1.15
$a'(F)$	0.7	0.8
W' (MeV)	40.0	0.0
$R_N=LCO(F)$	9.2	9.2

Kim and Rasmussen, which include the effects of configuration mixing, were not used since the differential cross section is only slightly changed (as will be shown below) by the use of these admixed wave functions. Two sets of optical parameters based on fits to He³ scattering data were tried. The optical parameters which led to the best fit to the present angular distribution were adopted and are listed in Table VI. The intrinsic single-particle cross sections calculated with this set of optical parameters are shown in Fig. 3.

The calculated triton spectrum for a scattering angle of 140° is shown in Fig. 4. The levels are labeled with their spin J and the configuration from which they originate. Also shown in the same figure is the experimentally observed triton spectrum from the reaction at a scattering angle of 140°. The gross agreement between the two spectra is readily apparent. The ground-state doublet $h_{9/2}p_{1/2}^{-1}$ in the calculated spectrum corresponds quite well in position and cross section to the observed levels Nos. 0 and 1. (In the notation used to describe the configurations, the proton state is on the left and the neutron hole state is on the right.) It is also apparent that the observed levels Nos. 6, 8, 9, and 10 nearly match the calculated intensities and positions of levels in the $h_{9/2}p_{3/2}^{-1}$ cluster. However, the agreement be-

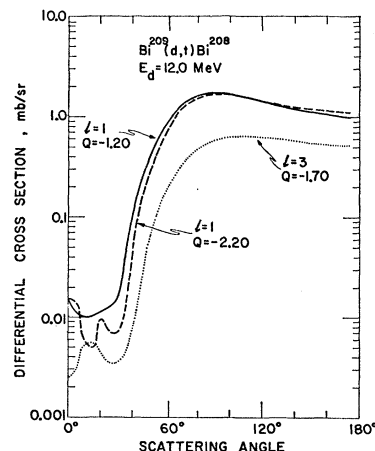


FIG. 3. Intrinsic single-particle differential cross sections calculated by the DWBA code. The parameters used for this calculation are given in Table VI.

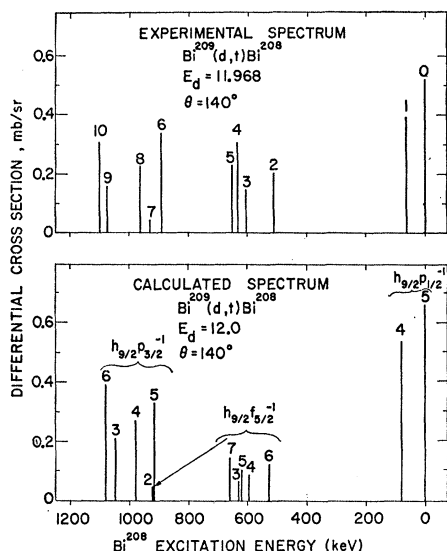


FIG. 4. A comparison between the experimental and calculated triton spectra from the $\text{Bi}^{209}(d,t)\text{Bi}^{208}$ reaction. In the experimental spectrum, the groups are labeled in sequence starting with 0 for the ground state. In the calculated spectrum, each group is labeled with its total angular momentum J as well as the proton-neutron hole configuration from which it originates.

tween the predicted levels in the $h_{9/2}f_{5/2}^{-1}$ cluster does not appear to be nearly as good since one level is missing in the experimental spectrum. By studying the intensities for these levels, one can see that it is quite likely that group No. 4 is composed of both the $J=5$ and $J=3$ states from the $h_{9/2}f_{5/2}^{-1}$ configuration. If this assumption is made, then all the levels in the three configurations have been accounted for.

It is interesting to see if the angular-distribution data support this interpretation. The angular distributions calculated by the DWBA code SALLY (Fig. 3) show that the differential cross section for the $l=3$ transition drops more rapidly than the $l=1$ cross section as the scattering angle is decreased from 100° to 60° . This result can be used to separate the $l=1$ levels from the $l=3$ levels in the experimental data. In Fig. 5, the change in differential cross section is displayed as the ratio of the observed cross section at 100° to that at 60° . In this figure the Q value of the level is plotted against this cross-section ratio. In the figure one can see that levels Nos. 2, 3, 4, 5, and 7 lie to the right of the line connecting levels Nos. 0 and 1 with the cluster made up of levels Nos. 6, 8, 9, and 10. This larger ratio for levels Nos. 2, 3, 4, 5, and 7 indicates that the l values corresponding to these levels is larger than the l value of the other levels. This behavior is completely consistent with identifying groups Nos. 2, 3, 4, 5, and 7 as belonging to the $h_{9/2}f_{5/2}^{-1}$ configuration and groups Nos. 6, 8, 9, and 10 as belonging to the $h_{9/2}p_{3/2}^{-1}$ configuration.

A visual study of the excitation energies and cross sections in the experimental and theoretical spectra (Fig. 4) quickly suggests spin assignments for the observed

levels. These spin assignments are included in Table III. However, the question now arises as to whether a more detailed study of the observed differential cross sections will support these spin assignments. Such a detailed examination cannot easily be made since the intrinsic single-particle cross section ϕ_l is not known accurately. This difficulty can be illustrated more precisely by writing the theoretical expression¹⁵ for the differential cross section in the form

$$\frac{d\sigma}{d\Omega} = \frac{(2J_B+1)}{(2J_A+1)} \sum_l S_l \phi_l(\theta).$$

In this notation J_A and J_B are the total angular momentum of the target and residual nucleus, respectively, S_l is the spectroscopic factor which contains the information on nuclear structure, and $\phi_l(\theta)$ is the intrinsic single-particle cross section. If S_l and ϕ_l were constants within a given configuration, it would be an easy matter to extract the spin J_B from the experimental data. In the present situation S_l is very nearly 1 or 0, as shown by the calculations of Kim and Rasmussen. This means that uncertainties in the wave functions will not seriously hinder the assigning of spins. However, the remaining quantity ϕ_l is a function of the Q value of the reaction and will change from one level to the next within a given configuration. This uncertainty in the Q dependence of ϕ_l is the principal difficulty in assigning the final-state spins on the basis of the observed cross sections.

Two methods of obtaining the ϕ_l were tried, each of which leads to a slightly different set of J values for the Bi^{208} levels. This lack of uniqueness is unfortunate but not too surprising since both methods of obtaining the

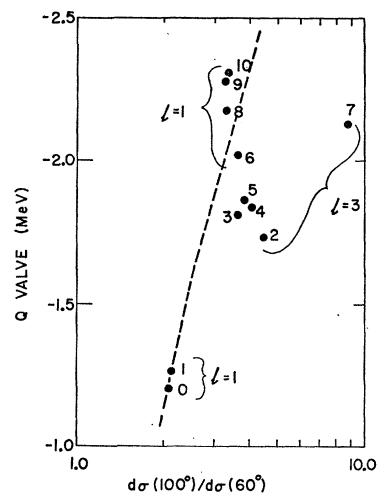


FIG. 5. The change in differential cross section displayed as the ratio of the observed cross section at 100° to that at 60° . The Q values of the levels are plotted against this ratio. The points are labeled with the identifying numbers used in Table III.

¹⁵ G. R. Satchler, Ann. Phys. (N. Y.) 3, 275 (1958).

ϕ_l are somewhat open to question. The first method is to use the DWBA calculations, as was done to obtain the calculated spectrum in Fig. 4. The differential cross sections calculated in this way are presented in Table VII in the column labeled theory 1. The spin assignments used are those given by Kim and Rasmussen. To simplify comparison between the various sets of cross sections in this table, the cross sections of all levels arising from one configuration have been normalized such that their sum is unity. In making these calculations, ϕ_l has been assumed to be a logarithmic function of Q for the purpose of extrapolating between the points calculated by the DWBA code. The first column gives the experimental cross sections which have been normalized in the same way. A comparison between the columns labeled experiment and theory 1 shows reasonably good agreement in the relative magnitude except for levels Nos. 6 and 10. Somewhat better agreement is achieved if the spins assigned to these levels are reversed. However, this reversal may not be correct as will be seen below when another set of ϕ_l is used to calculate the cross sections.

A second method of obtaining the intrinsic single-particle cross section ϕ_l leads to nearly the same values of J for the observed levels. This method uses the experimental data directly. By combining the yields of all levels in the $h_{9/2}p_{1/2}^{-1}$, $h_{9/2}f_{5/2}^{-1}$, and $h_{9/2}p_{3/2}^{-1}$ clusters and by assuming that these are good single-hole levels, experimental values for the intrinsic-particle cross sections can be obtained as a function of l , Q , and θ . These results are shown in Fig. 6. The ordinate is the sum of the differential cross sections for all levels from one configuration divided by $(2j+1)$, where $j = \frac{1}{2}, \frac{5}{2}$,

TABLE VII. A comparison of the experimental differential cross sections with theoretical differential cross sections which have been calculated by three different methods.

Level	$d\sigma/d\Omega^a$ ($\theta = 140^\circ$)				J	Configuration
	Experiment	Theory 1 ^b	Theory 2 ^c	Theory 3 ^d		
0	0.569	0.548	0.560	0.560	5	$h_{9/2}p_{1/2}^{-1}$
1	0.431	0.452	0.440	0.440	4	
2	0.216	0.220	0.261	0.252	6	$h_{9/2}f_{5/2}^{-1}$
3	0.159	0.150	0.154	0.159	4	
4	0.329	0.117	0.116	0.126	3	
		0.183	0.182	0.193	5	
5	0.246	0.249	0.237	0.225	7	$h_{9/2}p_{3/2}^{-1}$
7	0.050	0.080	0.050	0.047	2	
6	0.328	0.272	0.293	0.295	5	$h_{9/2}p_{3/2}^{-1}$
8	0.220	0.224	0.231	0.232	4	
9	0.153	0.176	0.167	0.158	3	
10	0.299	0.328	0.308	0.316	6	

^a The cross sections of all the levels have been normalized in such a way that their sum is unity for all the levels from a particular configuration.

^b The ϕ_l 's calculated by the DWBA code were used. No effects of configuration mixing are included.

^c The ϕ_l 's obtained empirically from the data were used. No effects of configuration mixing are included.

^d The ϕ_l 's obtained empirically from the data were used. Configuration-mixing effects are included by using the wave functions of Kim and Rasmussen.

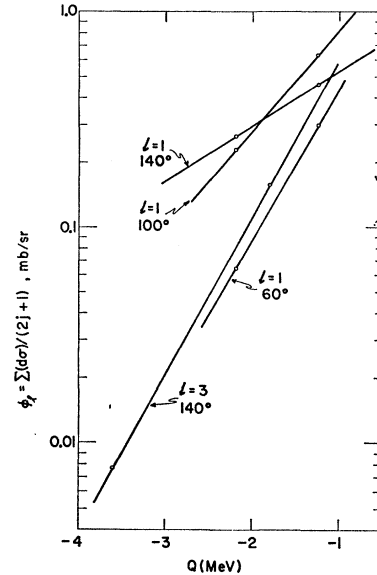


FIG. 6. The intrinsic single-particle cross sections which are derived from the experimental data. The sum of the differential cross sections of all levels from one configuration divided by $(2j+1)$ is plotted against the mean Q value of the levels from that configuration.

or $\frac{3}{2}$ depending on which configuration is being considered. Again, ϕ_l is assumed to vary logarithmically with Q . The single-particle cross sections obtained in this way were used to calculate the differential cross sections listed in the column labeled theory 2 in Table VII. This column is again to be compared with the experiment column. The disagreement for levels Nos. 6 and 10 is not so bad. However, now the agreement for levels Nos. 2 and 5 has been lost.

The effects of using the admixed wave functions of Kim and Rasmussen are shown in the column labeled theory 3. The same ϕ_l 's were used as in theory 2. There is some change, but it is smaller than the differences produced by the two choices of ϕ_l .

Although perfect agreement between the observed and theoretical cross sections has not been obtained, the few discrepancies which are found are probably not significant. Possibly these discrepancies are due to an improper set of intrinsic single-particle cross sections ϕ_l . Therefore, one can conclude that the calculated excitation energies of the levels in Bi²⁰⁸ made by Kim and Rasmussen are fairly good, and that the spin assignments of the observed level made in accordance with these theoretical calculations are probably correct.

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